MATH 512 HOMEWORK 1

Due Friday, Feb 15.

Problem 1. *Show that I is normal if and only if the dual filter is closed under diagonal intersections.*

Problem 2. *Suppose that U is a κ-complete normal ultrafilter on κ. Show that U contains all clubs of κ, and that every set in U is stationary.*

Problem 3. *Let κ be a regular uncountable cardinal and $\kappa < \lambda$. Show that $\{x \in \mathcal{P}_\kappa(\lambda) \mid x \cap \kappa \in \kappa\}$ is a club in $\mathcal{P}_\kappa(\lambda)$. Show that the club filter of $\mathcal{P}_\kappa(\lambda)$ is closed under diagonal intersections. Suppose that $\lambda < \tau$. Show that:*

- *if S is stationary in $\mathcal{P}_\kappa(\lambda)$, then $\{x \in \mathcal{P}_\kappa(\tau) \mid x \cap \lambda \in S\}$ is stationary in $\mathcal{P}_\kappa(\tau)$,*
- *if S^* is stationary in $\mathcal{P}_\kappa(\tau)$, then $\{x \cap \lambda \mid x \in S^*\}$ is stationary in $\mathcal{P}_\kappa(\lambda)$,*

Problem 4. *Let $V \subset V[G]$ and suppose that $V \models$ "C is club in κ". Show that $V[G] \models$ "C is club in κ" (even if κ is no longer a cardinal)*

Problem 5. *Let P_S be the poset to shoot a club through S, where S is a stationary subset of ω_1. Show that every stationary subset of S remains stationary in the generic extension.*

Recall that $\langle C_\alpha \mid \alpha \in \mathrm{Lim}(\kappa^+)\rangle$ is a $\square_\kappa$ sequence iff:

(1) each C_α is a club subset of α,
(2) for each α, if $\mathrm{cf}(\alpha) < \kappa$, then $o.t.(C_\alpha) < \kappa$,
(3) for each α, if $\beta \in \mathrm{Lim}(C_\alpha)$, then $C_\alpha \cap \beta = C_\beta$.

Problem 6. *Suppose that $\langle C_\alpha \mid \alpha \in Lim(\kappa^+)\rangle$ is a $\square_\kappa$ sequence. Show that there is no club $C \subset \kappa$ such that for all α, $C \cap \alpha = C_\alpha$.*

Hint: look at the order type of initial segments of such a C.